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GROWTH OF A BUBBLE AT A HEATED SURFACE IN A POOL OF LIQUID METAL

S. G. BANKOFF and H. K. CHOI
Chemical Engineering Department, Northwestern University, Evanston, Illinois, U.S.A.

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Abstract—A theoretical investigation of the initial vapor bubble growth from a heated surface wall in a pool of liquid sodium is reported. The analysis assumes the bubble to have the shape of a spherical sector, at the base of which a thin liquid microlayer is retained on the heating surface. The effects of time-and-space dependent heat conduction in the solid, microlayer vaporization, and non-equilibrium condensation on the bubble upper surface are considered. A two-term expression for the bubble growth rate is obtained by a collocation procedure. Calculated results predict the growth of the bubble on a heated surface as a function of the heat flux, the external pressure and the thermophysical properties of the liquid and solid. An expression due to Ruckenstein, modified to take into account the effective contact angle, is used to determine the departure bubble diameter. This turns out to be sensitive to the contact angle, and less strongly influenced by the bubble drag coefficient. The growth is initially inertia-controlled, but heat-transfer effects become significant before departure. In this pressure range (0·1–1 atm) the presence of inert gas, by reducing the effective accommodation coefficient for condensation, increases bubble growth rate moderately.

NOMENCLATURE

A,	constant, equation (3);
a_1, a_2,	constants, equation (28);
b_0, b_1, b_2,	constants, equation (31);
C_d,	drag coefficient;
C_p,	specific heat;
D_d,	departure diameter;
g,	acceleration due to gravity;
h_{fg},	latent heat of vaporization;
k,	thermal conductivity;
M,	molecular weight;
P,	pressure;
$\hat{q}$,	radial average wall heat flux;
$\bar{q}$,	steady-state wall-heat flux;
q_c,	condensing heat flux;
R,	bubble radius;
R_g,	gas constant;
r,	radial distance;
t,	time;
T,	temperature;
u_b,	bubble rise velocity.

Greek symbols

α,	thermal diffusivity;
δ,	boundary layer thickness;
ε,	accommodation coefficient;
ρ,	density;
σ,	surface tension;
τ,	delay time, equation (26);
θ,	contact angle.

Subscripts

f,	liquid;
g,	gas;
s,	saturation;
v,	vapor;
w,	wall.

1. INTRODUCTION

BUBBLE growth in the alkali liquid metals, such as sodium at pressures near atmospheric, tends to be inertia-controlled, in view of the low vapor density and high thermal conductivity of the liquid, together with the relatively large superheats. The present work deals with the growth of a vapor bubble in a pool of liquid metal initially at saturation temperature in contact with a heated solid surface. We do not consider here the details of the nucleation process, and neglect the heating of the liquid prior to the formation of a bubble. On the other hand, the formation is realistic for liquid-metal pool boiling in that inertial effects, microlayer vaporization, polar condensation, non-equilibrium vaporization, and the thermal properties of the solid surface are all taken into account.

2. PREVIOUS WORK

The equation of motion of a spherically symmetric flow field surrounding a growing vapor bubble is given by the Rayleigh [1] equation:

$$R\ddot{R}+\tfrac{3}{2}\dot{R}^2=\frac{P_v-P_s}{\rho_f} \tag{1}$$

where the viscous and surface tension terms have been neglected. If it is further assumed that the pressure within the bubble is constant, and the initial radius is zero, the solution is of the form

$$R=At. \tag{2}$$

On substitution into equation (1), one obtains

$$A^2=\frac{2(P_v-P_s)}{3\rho_f}. \tag{3}$$

Suppose that the liquid is initially uniformly superheated to a temperature, $\bar{T}_w$. Then, if the bubble wall temperature, T_b, is constant and equal to $\bar{T}_w$, implying negligible cooling due to vaporization, and if the

pressure within the bubble is equal to the equilibrium vapor pressure at the wall temperature, an approximate expression for the proportionality constant is given by applying the differential form of the Clausius–Clapeyron equation:

$$A = \left(\frac{2\rho_g h_{fg}\Delta T}{3\rho_f T_s}\right)^{1/2}$$
$$\Delta T = \bar{T}_w - T_s. \tag{4}$$

At large times, however, the cooling effect will be significant, and will eventually control the bubble growth.

Cooper [2] and Cooper and Lloyd [3] demonstrated that at low pressures the liquid microlayer between the solid surface and the growing bubble could be a significant factor in bubble growth on a heated surface. Sernas and Hooper [4] showed, by means of streak photographs of rapid bubble growth, that the bubble has a hemispherical shape in its early growth. Deane and Rohsenow [5] assumed that the principal source of vapor for liquid metal bubble growth is the liquid microlayer, and estimated that only about 10 per cent of the microlayer is vaporized during the bubble life on the wall. They constructed a model for the mechanism of nucleate boiling of a liquid metal on a solid heating surface by considering the effect of microlayer evaporation, bubble departure and temperature recovery time as a periodic process. The departure radius and time were calculated by means of a bubble departure criterion for nonmetallic liquids, equation [5], together with the numerical solution of Wichner and Hoffman [6] of the equations for rapid growth of an isolated bubble in a superheated liquid. This correlation, recommended by Cole and by Rohsenow [7, 8] for the bubble departure diameter in water, was

$$\left(\frac{g(\rho_f-\rho_g)D_d^2}{\sigma}\right)^{1/2} = 1{\cdot}5\times10^{-4}\left(\frac{C_{pf}\rho_f T_s}{\rho_g h_{fg}}\right)^{5/4}. \tag{5}$$

Theofanous *et al.* [9] obtained numerical solutions for rapid bubble growth in a pool of uniformly-superheated liquid, taking into account the effects of surface tension, liquid inertia, heat conduction in the liquid and non-equilibrium effects at the liquid–vapor interface.

Bobrovich *et al.* [10] (as reported in Subbotin *et al.* [11]) obtained experimental data on the departure diameter of fast-growing vapor bubbles in water, ethyl alcohol, benzene and potassium boiling at subatmospheric pressures. The potassium data can be closely represented by a simplified balance in which surface tension forces have been omitted.

$$D_d = \frac{\dot{D}^2}{g}. \tag{6}$$

Roll and Myers [12] likewise omitted surface effects in their force balance, obtaining

$$\frac{\pi D_d^3}{6}g(\rho_f-\rho_g) - \frac{C_D\pi D_d^2\rho_f u_b^2}{8} - \frac{\pi D_d^3\rho_f}{12}\left(\tfrac{11}{8}\right)\frac{du_b}{dt} = 0 \tag{7}$$

where buoyancy, drag, and acceleration forces are represented, and $u_b = dD_d/dt$ is the translational velocity of the spherical bubble.

Ruckenstein [13] obtained a similar expression for the departure diameter by considering only the buoyancy and drag forces:

$$\frac{\pi D_d^3 g(\rho_f-\rho_g)}{6} = C_D\frac{\pi D_d^2}{4}\rho_f\frac{u_b^2}{2} \tag{8}$$

where again $u_b = dD_d/dt$. We now consider extensions when the bubble does not grow at zero contact angle.

If the bubble shape at departure is hemispherical, the departure diameter becomes

$$\tfrac{2}{3}\pi R_d^3 g(\rho_f-\rho_g) = C_D\pi R_d^2\rho_f\frac{\dot{R}_d^2}{2} \tag{9}$$

or

$$C_D\dot{R}_d^2 - \tfrac{4}{3}gR_d = 0. \tag{10}$$

More generally, if the bubble shape at departure is a spherical segment with contact angle, θ, the departure criterion will be

$$\frac{\pi R_d^3}{3}(2+3\cos\theta-\cos^3\theta)g(\rho_f-\rho_g) = C_D\pi R_d^2\rho_f\frac{R_d^2(1+\cos\theta)^2}{2} \tag{11}$$

or

$$C_D\dot{R}_d^2 - \frac{(2+3\cos\theta-\cos^3\theta)}{(1+\cos\theta)^2}gR_d = 0. \tag{12}$$

This extension of the Ruckenstein expression is used in the present work.

To our present knowledge, no experimental data on bubble growth rates from a heated surface in a liquid-metal pool exist, with which to compare the theory in this paper since the completion of this work. Dwyer *et al.* [15, 16] have independently arrived at theoretical solutions to a similar problem, involving, however, the assumptions that dryout of the microlayer occurs quite rapidly, and that the liquid surrounding the bubble is uniformly superheated at the heating surface temperature. The predictions of the two theories can therefore not be directly compared.

3. FORMULATION OF THE PROBLEM

Consider a growing vapor bubble on a horizontal heated surface in a pool of liquid metal which is everywhere at rest and at the saturation temperature at the time bubble starts to grow (Fig. 1). We assume the bubble to be a spherical segment with a constant contact angle over time intervals of interest and the flow field to be purely radial. This implies the neglect of bubble distortion close to the wall due to motion of bubble center away from wall, and of distortion of the flow field due to the zero velocity of the solid surface. It is further assumed that a liquid microlayer separates the vapor at the base of the bubble from the solid, which offers negligible thermal resistance, remains intact and stationary during the bubble lifetime, and furnishes essentially all of the vapor entering the bubble. This neglects vaporization from liquid surrounding the bubble, which is expected to be small compared to microlayer vaporization. As the bubble grows, fresh

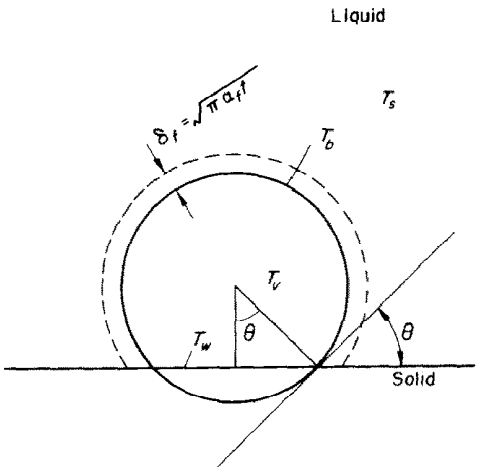

FIG. 1(a). Geometry of bubble growth.

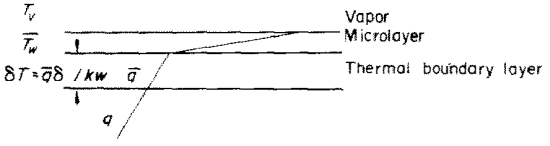

FIG. 1(b). Heat-transfer resistance through the microlayer.

microlayer is uncovered continuously, and a thermal boundary layer builds up in the solid surface below the microlayer. Inserting now the Clausius–Clapeyron equation into equation (1), one obtains an approximate relationship between the vapor superheat and the bubble motion:

$$R\ddot{R} + \tfrac{3}{2}\dot{R}^2 = \frac{\rho_g h_{fg}}{\rho_f T_s}(T_v - T_s) \tag{13}$$

where the vapor temperature will generally lie between the wall temperature, $\bar{T}_w$, and the saturation temperature, T_s.

It is now assumed that the evaporation of vapor into the bubble occurs entirely from the microlayer at the base of the bubble, while condensation of the vapor occurs continuously at the bubble surface. The condensing heat flux is assumed to be uniform, since the bulk liquid temperature and vapor temperature are uniform at every instant. The resistance to heat flow consists principally of a thin thermal boundary layer in a liquid in series with an interfacial resistance to molecular condensation. Assuming that the heat flow is continuous through the liquid–vapor interface, one obtains

$$\begin{aligned} q_c &= \frac{k_f}{\sqrt{(\pi \alpha_f t)}}(T_b - T_s) \\ &= \varepsilon \sqrt{\left(\frac{M}{2\pi R_g T_v}\right)} \frac{\rho_g h_{fg}}{T_s}(T_v - T_s) h_{fg}. \end{aligned} \tag{14}$$

Upon eliminating the bubble wall temperature, T_b, one obtains

$$q_c = \frac{T_v - T_s}{\dfrac{T_s}{\varepsilon \rho_g h_{fg}^2}\left(\dfrac{2\pi R_g T_v}{M}\right)^{1/2} + \dfrac{(\pi \alpha_f t)^{1/2}}{k_f}} \tag{15}$$

where the second term in the denominator very quickly becomes large compared to the first term in the absence of inert gas. If inert gas is present in any substantial amount, the accommodation coefficient, ε, will be much less than one, with a corresponding reduction in bubble wall condensation. Since this effect is not appreciable for evaporation, this would imply that bubble growth in the presence of inert gases may be substantially faster than in their absence. In the absence of such effects, a reasonable approximation over the entire range is then:

$$q_c = \frac{k_f}{\sqrt{(\pi \alpha_f)}} \frac{T_v - T_s}{\sqrt{t}}. \tag{16}$$

To determine the local heat flux from the wall into the bubble, we assume that the heat flux and the wall temperature are both position- and time-dependent:

$$\begin{aligned} q &= q(r, t) \\ T_w &= T_w(r, t). \end{aligned} \tag{17}$$

Initially, the heat flow and the wall temperature are assumed to be steady-state values:

$$\begin{aligned} \bar{q} &= q(r, 0) \\ \bar{T}_w &= T_w(r, 0). \end{aligned} \tag{18}$$

As the bubble grows, vapor is generated by evaporation of the liquid microlayer remaining next to the solid surface. We assume that the thermal resistance of the microlayer is negligible, in view of its large thermal conductivity and small thickness. A uniform temperature gradient in the wall is postulated initially. The thickness of the thermal boundary layer in the solid under the bubble will depend upon the time of exposure of the surface element to vapor, and hence is a function of radial position as well as time. The temperature driving force across this boundary layer of thickness $\delta_w(r, t)$, is then:

$$\Delta T_w(r, t) = \bar{T}_w + \bar{q} \frac{\delta_w(r, t)}{k_w} - T_w(r, t). \tag{19}$$

The instantaneous local heat flux is

$$q(r, t) = \frac{k_w}{\delta_w(r, t)} \Delta T_w(r, t). \tag{20}$$

Hence

$$q(r, t) = \bar{q} + \frac{k_w}{\delta_w(r, t)}(\bar{T}_w - T_w(r, t)). \tag{21}$$

With the assumption of a uniform vapor temperature the heat flux across the gas–liquid interface becomes

$$q(r, t) = \varepsilon \sqrt{\left(\frac{M}{2\pi R_g T_v}\right)} \frac{\rho_g h_{fg}^2}{T_s}(T_w(r, t) - T_v(t)) \tag{22}$$

where ε is the accommodation coefficient. It is possible to eliminate T_w from equations (20) and (22), and with the assumption that the interfacial resistance is small compared to the thermal boundary layer resistance

$$\frac{\delta_w(r, t)}{k_w} \gg \frac{1}{\varepsilon} \sqrt{\left(\frac{2\pi R_g T_v}{M}\right)} \frac{T_s}{\rho_g h_{fg}^2} \tag{23}$$

for $\varepsilon = 1$, one obtains approximately

$$q(r, t) = \bar{q} + \frac{k_w}{\delta_w(r, t)}(\bar{T}_w - T_v). \tag{24}$$

The thermal boundary-layer thickness is a function of both time and position and it may be written in the parametric form:

$$\delta_w(r,t) = 2\sqrt{(\alpha_w(t-\tau))} \tag{25}$$

$$R(\tau) = \frac{r}{\sin\theta} \tag{26}$$

where τ is the time at which the surface at position r came into contact with the vapor. One can eliminate the radial dependence by defining $\hat{q}$:

$$\hat{q}(t) = \frac{1}{\pi R^2 \sin^2\theta}\int_0^{R\sin\theta}\left\{\bar{q} + \frac{k_w(\bar{T}_w - T_v)}{2\sqrt{(\alpha_w(t-\tau))}}\right\}\mathrm{d}(\pi r^2). \tag{27}$$

Approximating the bubble radius by the two-term expression,

$$R = a_1 t + a_2 t^2 \tag{28}$$

where a_1 and a_2 are constants to be determined, the initial condition, $R(0) = 0$, is satisfied.

Now letting $\tau' = \tau/t$ and substituting equation (28) into (27), one gets

$$\hat{q} = \bar{q} + \frac{k_w(\bar{T}_w - T_v)t}{R^2\sqrt{(\alpha_w t)}} \times \int_0^1 \frac{(a_1 t\tau' + a_2 t^2\tau'^2)(a_1 + 2a_2 t\tau')\mathrm{d}\tau'}{\sqrt{(1-\tau')}}. \tag{29}$$

Integrating and rearranging equation (29), one obtains

$$\hat{q} = \bar{q} + \frac{k_w}{\sqrt{(\alpha_w t)}}\frac{(\bar{T}_w - T_v)}{(a_1 + a_2 t)^2} \times \left\{\tfrac{4}{3}a_1^2 + \tfrac{16}{5}a_1 a_2 t + \tfrac{64}{35}a_2^2 t^2\right\}. \tag{30}$$

Substituting equation (28) into (13), we have

$$T_v = b_0 + b_1 t + b_2 t^2 \tag{31}$$

where $b_0 = \dfrac{1{\cdot}5\rho_f T_s}{\rho_g h_{fg}} a_1^2 + T_s$

$$b_1 = \frac{8\rho_f T_s}{\rho_g h_{fg}} a_1 a_2$$

$$b_2 = \frac{8\rho_f T_s}{\rho_g h_{fg}} a_2^2.$$

Since the surface area of the bubble

$$= 2\pi R^2(1+\cos\theta), \tag{32}$$

the base area

$$= \pi R^2 \sin^2\theta, \tag{33}$$

and the volume of the bubble

$$= \frac{\pi R^3}{3}(2 + 3\cos\theta - \cos^3\theta), \tag{34}$$

a simple heat balance requires that

$$\hat{q}(\pi R^2\sin^2\theta) = q_c 2\pi R^2(1+\cos\theta) + \rho_g h_{fg}(2 + 3\cos\theta - \cos^3\theta)\dot{R}\pi R^2. \tag{35}$$

Upon substituting equations (16), (29) and (20) into (35) and rearranging, we finally have

$$\bar{q} + \frac{k_w}{\sqrt{(\alpha_w t)}}\frac{(\bar{T}_w - T_v)}{(a_1 + a_2 t)^2}\left(\tfrac{4}{3}a_1^2 + \tfrac{16}{5}a_1 a_2 t + \tfrac{64}{35}a_2^2 t^2\right) - \frac{2k_f(T_v - T_s)}{\sqrt{(\pi\alpha_f t)}}\left(\frac{1+\cos\theta}{\sin^2\theta}\right) - \frac{\rho_g h_{fg}}{\sin^2\theta}(a_1 + 2a_2 t)(2 + 3\cos\theta - \cos^3\theta) = 0. \tag{36}$$

In view of the approximation implied by equation (28), equation (36) cannot be satisfied at all times. We therefore choose a_1, a_2 such that it is satisfied at two characteristic times, t_c, and $t_c/2$. We identify t_c with the estimate of the bubble departure time given by Deane and Rohsenow [5]. Because of the considerable difference in assumptions, we cannot expect that t_c will be very close to the departure time for the bubbles considered in this work.

To calculate $\bar{T}_w$, the experimental relation by Subbotin *et al.* [10] for stable boiling of sodium, in the absence of argon, on stainless steel, nickel, chromium, or copper at pressures between about 0·1 and 1 atm was used:

$$\bar{T}_w - T_s = 4\tilde{q}^{1/3} \tag{37}$$

where $\tilde{q}$ is the time-average heat flux, in kcal/m²/h.

The modified Ruckenstein criterion, equation (10), was used to determine departure time and radius.

4. RESULTS

Sodium bubble radius–time curves for various contact angles with $t_c = 0{\cdot}003$ s, $\Delta T = 10{\cdot}4$°C and $\bar{q} = 0$ (uniform initial solid temperature) are shown in Figs. 2 and 3 for $P_s = 1{\cdot}0$ and 0·1316 atm. The departure lines give the loci of departure points for two drag coefficients, $C_D = 0{\cdot}5$ and 1·0. At the higher pressure the growth curves deviate appreciably from straight lines before departure, indicating that the heat diffusion terms are significant, while the drag coefficient does not strongly affect the departure radius. At the lower pressure, bubble growth is considerably more rapid in view of the reduced vapor density, giving rise to inertia-controlled growth and departure. These figures show a strong dependence on contact angle, with very slow growth for contact angles below 30°. One should realize that the effective contact angle, as determined by the ratio of bubble height to base diameter, may be considerably larger than the equilibrium contact angle for these flattened bubbles. The assumption that all of the vapor is generated at the bubble base is probably acceptable for the fast-growing bubbles, but tends to break down for slower growth rates. These calculations show an equilibrium bubble configuration at $\theta \sim 10$–15°, where microlayer evaporation is balanced by surface condensation, but this neglects heat conduction from the solid wall to the liquid surrounding the bubble.

Surprisingly, the calculated bubble growth rate is not very sensitive to the details of the initial wall temperature distribution. This can be seen from Fig. 4, where the initial wall temperature has been nearly doubled compared to the assumed wall temperature in Fig. 2. Similarly, a non-isothermal wall, corresponding to an initial steady flux of 2×10^5 kcal/m²/h, shows a small increase in bubble growth rate (Fig. 5). This implies that, although the microlayer vaporization rate is somewhat increased by the higher temperature driving force, the vapor temperature and condensation rate also increase, leading to a partial cancellation of effects. A much more significant variable is the bubble shape, since this determines the rate at which new

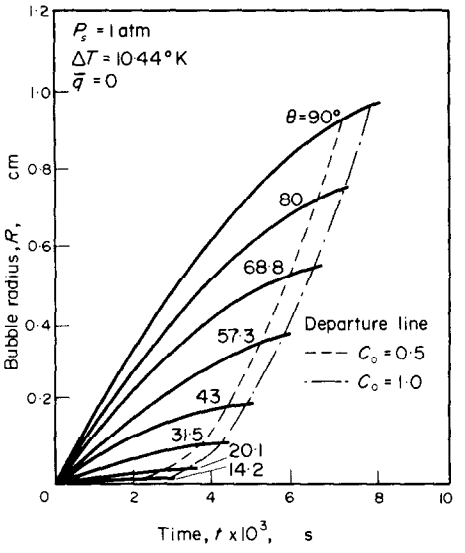

FIG. 2. Bubble radius vs time at various contact angles and bubble departure points for $C_D = 0{\cdot}5$ and 1 at $P_s = 1$ atm, $\Delta T = 10{\cdot}44°K$, $\bar{q} = 0$.

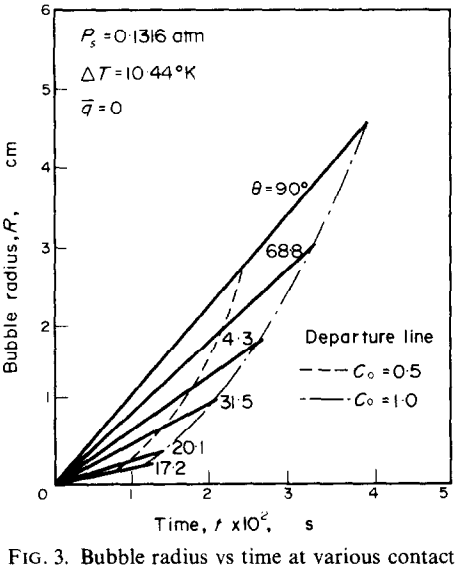

FIG. 3. Bubble radius vs time at various contact angles and bubble departure points for $C_D = 0{\cdot}5$ and 1 at $P_s = 0{\cdot}1316$ atm, $\Delta T = 10{\cdot}44°K$, $\bar{q} = 0$.

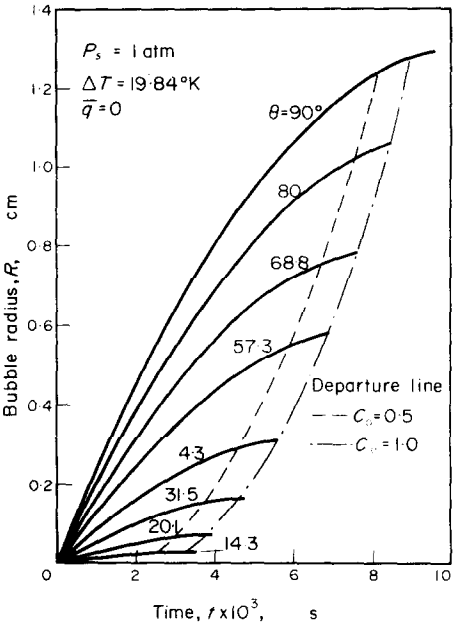

FIG. 4. Bubble radius vs time at various contact angles and bubble departure points for $C_D = 0{\cdot}5$ and 1 at $P_s = 1$ atm, $\Delta T = 19{\cdot}84°K$, $\bar{q} = 0$.

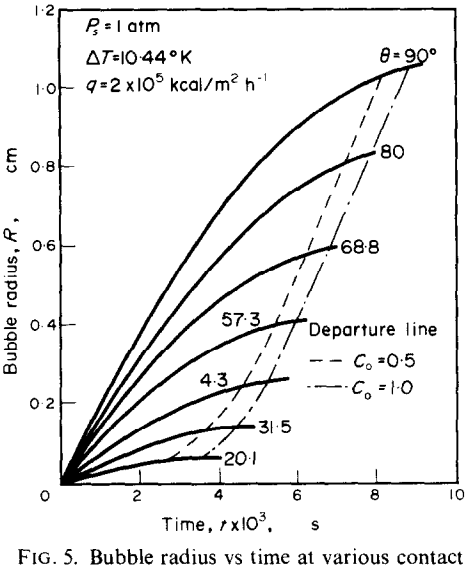

FIG. 5. Bubble radius vs time at various contact angles and bubble departure points for $C_D = 0{\cdot}5$ and 1 at $P_s = 1$ atm, $\Delta T = 10{\cdot}44°K$, $q = 2 \times 10^5$ kcal/m²/h.

microlayer liquid is exposed, from which the bulk of the vapor arises. The vapor temperature is, in fact, a weak function of time [14], and is essentially constant for inertia-controlled growth. The vapor superheat is 5°C or less, which is to be expected, since the condensation area is always at least twice the bubble base area.

Figures 6 and 7 show that the accommodation coefficient, ε, has only a minor effect under these conditions, indicating that the condensing flux, q_c is relatively small. A comparison is also shown with the growth rate at a high superheat ($\Delta T = 151{\cdot}5°K$) calculated by Theofanous *et al.* [9] for an isolated sodium bubble in a pool of uniformly-superheated liquid. As expected, this growth rate is somewhat larger than for a bubble attached to a wall at the same initial superheat, but with no superheating of the liquid next to the wall.

5. DISCUSSION

It is seen that the calculated departure time, t_d, varies considerably with contact angle, pressure and heat flux and is, in general, different from that calculated by Deane and Rohsenow [5], t_c. The models have important differences, however, in the departure criterion, the assumed heat transfer coefficient from the microlayer, the initial surface wall superheat and the initial temperature distribution in the solid. We therefore used t_c only a characteristic time at which to satisfy the

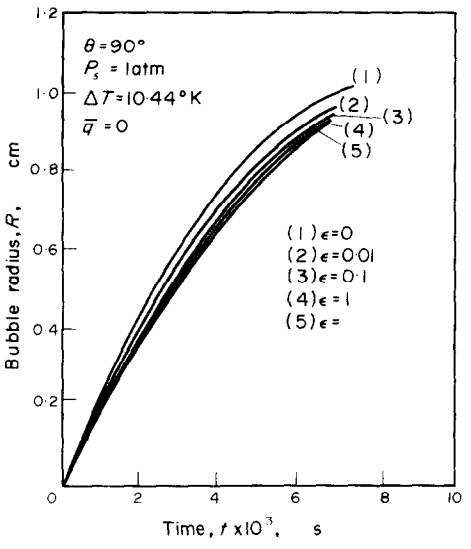

FIG. 6. Bubble growth for different accommodation coefficients at $P_s = 1$ atm, $\theta = 90°$, $\bar{q} = 0$.

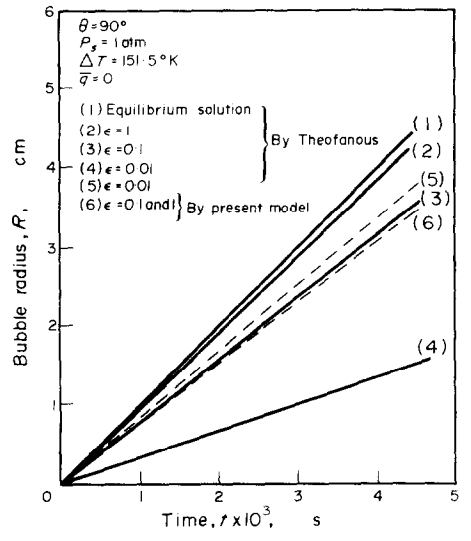

FIG. 7. Comparison of present theory with Theofanous *et al.* [9] calculations for spherical bubble at 1 atm, $\Delta T = 151{\cdot}5°K$, $\theta = 90°$.

approximate equation (36) exactly. In addition, the growth curves satisfy this equation exactly at $t_c/2$, and the initial condition at $t = 0$. Since the curvature is everywhere small, this collocation procedure results in a small error at every point. It has been shown that the growth curves are quite insensitive, even for $t \leqslant 3t_c$, to the choice of the interior collocation point. The quadratic term in equation (28) is small for these fast-growing bubbles, and represents a deviation from linearity for the short times prior to departure. At still higher pressures, this term would become quite important. We have assumed one-dimensional flow under the bubble, which seems reasonable in view of the short contact times. A more serious assumption is implied in the neglect of liquid superheat prior to bubble growth. These results may be somewhat low, therefore, although the comparison with the results of Theofanous *et al.* [9] indicates that this correction should be small.

The general conclusions from this work point to the predominant effects of the liquid microlayer and of the flattening of the bubble in determining bubble growth rates and departure times in liquid-metal pool boiling. The vapor superheat is only a few degrees, so that the condensation heat flux effects are relatively minor in the heat flux and pressure range tested. For the same reason, the effects of inert gas, as shown by the effective accommodation coefficient, are not here very important, although they would be expected to be very significant at still lower pressures.

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CROISSANCE D'UNE BULLE SUR LA PAROI CHAUDE D'UN RESERVOIR PLEIN DE METAL LIQUIDE

Résumé—On présente une étude théorique du développement d'une bulle de vapeur sur une paroi chauffée d'un réservoir de sodium liquide. L'analyse suppose que la bulle a la forme d'un secteur sphérique à la base duquel un film liquide très fin est retenu sur la surface chauffante. On a considéré les effets de la conduction thermique variable dans l'espace et dans le temps à l'intérieur du solide de la vaporisation du film liquide, et de la condensation en non-équilibre à la surface supérieure de la bulle. Une expression à deux termes est obtenue pour la vitesse de croissance de la bulle par une méthode de collocation. Les résultats calculés permettent de prévoir le développement de la bulle sur la surface chauffée en fonction du flux thermique, de la pression extérieure et des propriétés thermophysiques du liquide et du solide. On utilise une expression dûe à Ruckenstein, modifiée afin de tenir compte de l'angle de contact effectif, pour déterminer le diamètre de la bulle correspondant à son décrochement. Cette valeur se trouve être surtout sensible à l'angle de contact et moins fortement influencée par le coefficient de trainée de la bulle. La formation est d'abord controlée par les effets d'inertie, mais les effets de transfert thermique deviennent importants avant le décrochage. Dans ce domaine de pression (0,1–1 atm) la présence de gaz inertes, en réduisant le coefficient effectif d'accommodation pour la condensation, augmente modérément la vitesse de développement de la bulle.

DAS ANWACHSEN EINER BLASE AN DER HEIZFLÄCHE EINES MIT FLÜSSIGMETALL GEFÜLLTEN BEHÄLTERS

Zusammenfassung—In einer theoretischen Untersuchung wird über den Beginn des Blasenwachsens an einer beheizten Wand eines mit flüssigem Natrium gefüllten Behälters berichtet. Die Analyse geht aus von einer Dampfblase von der Form eines Kugelsektors, an dessen Basis eine dünne, flüssige Mikroschicht an der Heizfläche haftet. Die Einflüsse der zeit- und ortsabhängigen Wärmeleitung in der Heizwand, die Mikroschichtverdampfung und die Kondensation an der Blasenoberfläche werden berücksichtigt. Dabei ergibt sich ein zweigliedriger Ausdruck für die Blasenwachstumsgeschwindigkeit. Das Blasenwachstum erweist sich abhängig vom Wärmestrom, vom äußeren Druck, von den physikalischen Eigenschaften der Flüssigkeit und der Heizfläche. Zur Bestimmung des Blasen-abreißdurchmessers wird ein modifizierter Ausdruck von Ruckenstein herangezogen, der den wirksamen Kontaktwinkel einbezieht. Dieser Ausdruck erweist sich als abhängig vom Kontaktwinkel und weniger abhängig vom Blasennachlauf-Koeffizienten. Das Wachstum ist anfänglich trägheitsbestimmt, erst vor dem Abreißen werden Wärmeübergangseinflüsse wirksam. Im untersuchten Druckbereich (0,1 bis 1 bar) werden die Blasenwachstumsgeschwindigkeiten bei Anwesenheit von Inertgas etwas erhöht, da die effektiven Akkomodationskoeffizienten für Kondensation reduziert werden.

РОСТ ПУЗЫРЬКА НА НАГРЕТОЙ ПОВЕРХНОСТИ ПРИ КИПЕНИИ ЖИДКОГО МЕТАЛЛА В БОЛЬШОМ ОБЪЕМЕ

Аннотация — Описывается теоретическое исследование начального роста пузырька пара с нагретой поверхности стенки при кипении натрия в большом объеме. Предполагается, что пузырек имеет форму участка сферы, у основания которой на нагретой поверхнрсти сохраняется тонкий микрослой жидкости. Рассматриваются эффекты теплопроводности, зависящей от времени и пространства, в твердом теле, испарения микрослоя жидкости и неравновесной конденсации на верхней поверхности пузырька. Методом коллокации получено двухчленное выражение для скорости роста пузырька. Расчетные данные предсказывают рост пузырька на нагретой поверхности в зависимости от теплового потока, внешнего давления и теплофизических свойств жидкости и твердого тела. Для определения диаметра отрыва пузырька используется выражение по Рукенштейну, модифицированное для учета эффективного угла контакта. Оказывается, что диаметр отрыва пузырька сильно зависит от угла контакта и менее сильно, от коэффициента торможения пузырька. Вначале рост пузырька регулируется инерционными силами, но до его отрыва начинают сказываться эффекты теплообмена. В этом диапазоне давлений (0,1–1 атм) наличие инертного газа несколько повышает скорость роста пузырька за счет уменьшения эффективного коэффициента аккомодации при конденсации.